

- 2 The table gives information about the speeds, in kilometres per hour, of 80 motorbikes as each pass under a bridge.

Speed (s kilometres per hour)	Frequency
$40 < s \leq 50$	10
$50 < s \leq 60$	16
$60 < s \leq 70$	19
$70 < s \leq 80$	23
$80 < s \leq 90$	12

- (a) Write down the modal class.

$$70 < s \leq 80$$

(1)

- (b) Work out an estimate for the mean speed of the motorbikes as they pass under the bridge. Give your answer correct to 3 significant figures.

$$\begin{array}{r}
 45 \times 10 = 450 \\
 55 \times 16 = 880 \\
 65 \times 19 = 1235 \\
 75 \times 23 = 1725 \\
 85 \times 12 = 1020 \\
 \hline
 5310
 \end{array}$$

$$\begin{array}{r}
 5310 \div 80 \\
 = 66.375 \\
 \uparrow \\
 (3 \text{ sf})
 \end{array}$$

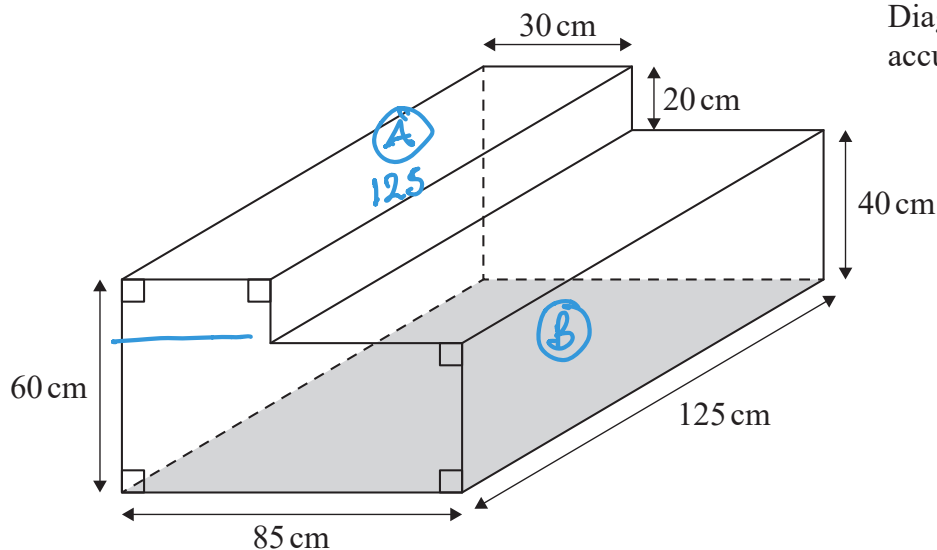
$$66.4 \text{ kilometres per hour}$$

(4)

(Total for Question 2 is 5 marks)



- 3 The diagram shows a container for water in the shape of a prism.



The rectangular base of the prism, shown shaded in the diagram, is horizontal. The container is completely full of water.

Tuah is going to use a pump to empty the water from the container so that the volume of water in the container decreases at a constant rate.

The pump starts to empty water from the container at 1030 and at 1200 the water level in the container has dropped by 20 cm.

Find the time at which all the water has been pumped out of the container.

$$\begin{aligned} \text{Vol}_A &= 30 \times 20 \times 125 \\ &= 75000 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Vol}_B &= 85 \times 40 \times 125 \\ &= 425000 \text{ cm}^3 \end{aligned}$$

A empties in 1 hour 30 mins

$$\text{so rate of flow} = \frac{75000}{90}$$

$$= 833.\bar{3} \text{ cm}^3/\text{min}$$

$$\begin{aligned} \text{Time}_B &= 425000 \div 833.\bar{3} \\ &= 510 \text{ mins} = 8 \text{ hrs } 30 \text{ mins} \end{aligned}$$

$$\begin{array}{ccc} & \xrightarrow{8 \text{ hrs}} & \\ 12:00 & & 20:00 \\ & \xrightarrow{30 \text{ mins}} & \\ & & 20:30 \end{array}$$

20:30

(Total for Question 3 is 4 marks)



4 $\mathcal{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$A = \{\text{odd numbers}\}$ 21 23 25 27 29

$B = \{\text{multiples of 3}\}$ 21 24 27

List the members of the set

(i) $A \cap B$

21, 27

(1)

(ii) $A \cup B$

21, 23, 24, 25, 27, 29

(1)

(Total for Question 4 is 2 marks)



5 (a) Factorise fully $15y^4 + 20uy^3$

$$\underline{5y^3(3y + 4u)} \quad (2)$$

(b) Solve $4 - 3x = \frac{5 - 8x}{4}$

Show clear algebraic working.

$\times 4$

$$4(4 - 3x) = 5 - 8x$$

$$16 - 12x = 5 - 8x$$

$$16 - 5 = -8x + 12x$$

$$11 = 4x$$

$$x = \frac{11}{4}$$

$$x = \underline{2.75} \quad (3)$$

(Total for Question 5 is 5 marks)

6 (a) Write 2 840 000 000 in standard form.

$$\underline{2.84 \times 10^9} \quad (1)$$

(b) Write 2.5×10^{-4} as an ordinary number.

$$\underline{0.00025} \quad (1)$$

(Total for Question 6 is 2 marks)



7 Chen invests 40 000 yuan in a fixed-term bond for 3 years.

The fixed-term bond pays compound interest at a rate of 3.5% each year.

1.035

(a) Work out the value of Chen's investment at the end of 3 years.

Give your answer to the nearest yuan.

$$40000 \times 1.035^3$$

$$= 44348.715$$

↑

44349

..... yuan
(3)

Wang invested P yuan.

The value of his investment decreased by 6.5% each year.

At the end of the first year, the value of Wang's investment was 30481 yuan.

(b) Work out the value of P .

$$100 - 6.5 = 93.5$$

$$\begin{array}{l} \div 93.5 \downarrow \\ 93.5\% = 30481 \\ 1\% = 326 \\ \times 100 \downarrow \\ 100\% = 32600 \end{array}$$

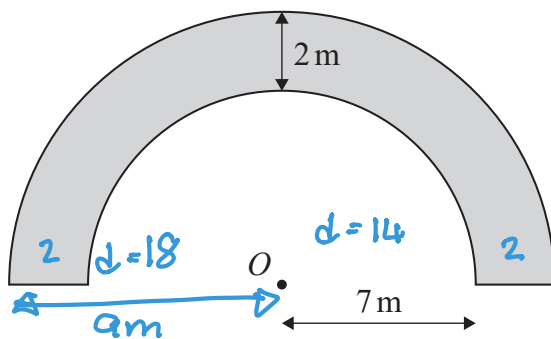
$P =$ 32600
.....
(3)

(Total for Question 7 is 6 marks)



8 The region, shown shaded in the diagram, is a path.

Diagram NOT accurately drawn



The boundary of the path is formed by two semicircles, with the same centre O , and two straight lines.

The inner semicircle has a radius of 7 metres.
The path has a width of 2 metres.

$$A = \pi r^2 \quad C = \pi d$$

Work out the perimeter of the path.
Give your answer correct to one decimal place.

$$\begin{aligned} & \frac{1}{2} \pi \times 18 + \frac{1}{2} \pi \times 14 + 2 + 2 \\ & = 54.2654 \dots \\ & \quad \uparrow \\ & \quad (1 \text{ dp}) \end{aligned}$$

54.3

..... m

(Total for Question 8 is 3 marks)



9 (a) Simplify $(2x^3y^5)^4$

$$2^4 x^{3 \times 4} y^{5 \times 4}$$

$$16x^{12}y^{20}$$

(2)

(b) (i) Factorise $x^2 + 5x - 36$

$$\begin{array}{l} 1, 36 \\ 2, 18 \\ 3, 12 \\ 4, 9 \end{array}$$

$$9 - 4$$

$$(x + 9)(x - 4)$$

$$(x + 9)(x - 4)$$

(2)

(ii) Hence, solve $x^2 + 5x - 36 = 0$

$$(x + 9)(x - 4) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ -9 & +4 \end{array}$$

$$x = -9 \quad x = 4$$

(1)

(Total for Question 9 is 5 marks)



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10 Here is isosceles triangle ABC .

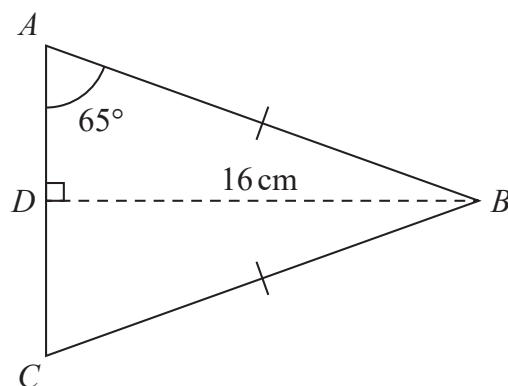
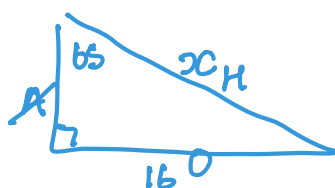


Diagram **NOT** accurately drawn

D is the midpoint of AC and $DB = 16$ cm.

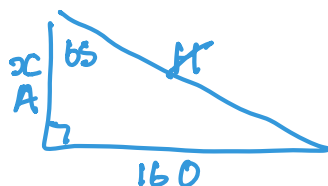
Angle $DAB = 65^\circ$

Work out the perimeter of triangle ABC .
Give your answer correct to one decimal place.



$$\sin 65 = \frac{16}{x}$$

$$x = \frac{16}{\sin 65} \\ = 17.654\dots$$



$$\tan 65 = \frac{16}{x}$$

$$x = \frac{16}{\tan 65} \\ = 7.4609\dots$$

$$\begin{aligned} \text{Perimeter} &= 2 \times 17.654 + 2 \times 7.4609 \\ &= 50.229\dots \\ &\quad \uparrow \\ &\quad (1 \text{ dp}) \end{aligned}$$

..... 50.2 cm

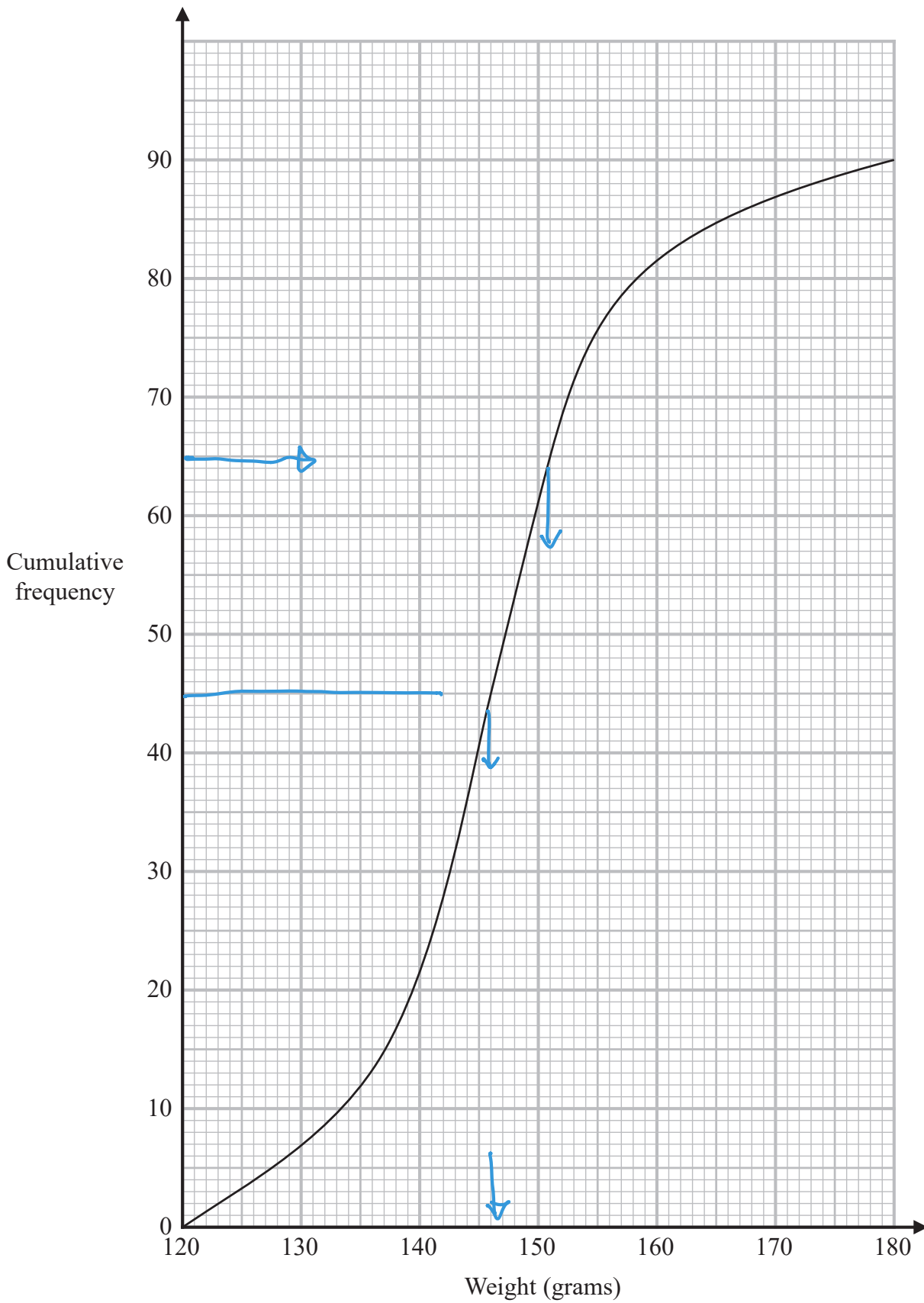
(Total for Question 10 is 4 marks)

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11 The cumulative frequency graph gives information about the weights, in grams, of 90 bags of sweets.



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(a) Find an estimate for the median of the weights of these bags of sweets.

.....146..... grams
(2)

Roberto sells the bags of sweets to raise money for charity.

Bags with a weight greater than d grams are labelled large bags and sold for 3.75 euros each bag.

The total amount of money he receives by selling all the large bags is 93.75 euros.

(b) Find the value of d .

$$93.75 \div 3.75 = 25 \text{ bags}$$

$$\text{so } 90 - 25 = 65$$

$d =$ 151.....
(3)

(Total for Question 11 is 5 marks)

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DO NOT WRITE IN THIS AREA



12 (a) Express $\frac{4}{x-2} - \frac{3}{x+1}$ as a single fraction.

Give your answer in its simplest form.

$$\begin{aligned} & \frac{4(x+1) - 3(x-2)}{(x-2)(x+1)} \\ &= \frac{4x+4-3x+6}{(x-2)(x+1)} \\ &= \frac{x+10}{(x-2)(x+1)} \end{aligned}$$

$$\frac{x+10}{(x-2)(x+1)} \quad (3)$$

(b) Expand and simplify $2x(x-5)(x-3)$

$$\begin{aligned} &= 2x(x^2-8x+15) \\ &= 2x^3-16x^2+30x \end{aligned}$$

$$2x^3-16x^2+30x \quad (3)$$

(Total for Question 12 is 6 marks)



- 13 Point A has coordinates $(5, 8)$
Point B has coordinates $(9, -4)$

(a) Work out the gradient of AB .

$$\begin{aligned}\text{gradient} &= \frac{-4-8}{9-5} \\ &= \frac{-12}{4} = -3\end{aligned}$$

.....
-3

(2)

The straight line L has equation $y = -4x + 5$

(b) Write down the gradient of a straight line that is perpendicular to L .

$$\begin{aligned}m_1 m_2 &= -1 \\ -4 \times m_2 &= -1 \\ m_2 &= \frac{-1}{-4} = \frac{1}{4}\end{aligned}$$

.....
 $\frac{1}{4}$

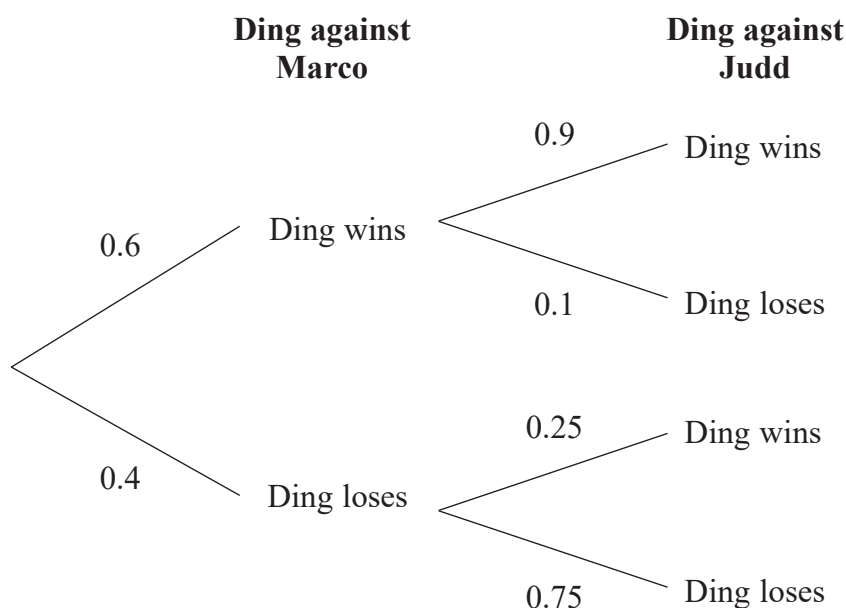
(1)

(Total for Question 13 is 3 marks)



- 14 Ding is going to play one game of snooker against each of two of his friends, Marco and Judd.

The probability tree diagram gives information about the probabilities that Ding will win or lose each of these two games.



- (a) Work out the probability that Ding will win both games.

$$P(WW) = 0.6 \times 0.9$$

0.54

(2)

- (b) Work out the probability that Ding will win exactly one of the games.

$$P(WL) = 0.6 \times 0.1 = 0.06$$

$$+ P(LW) = 0.4 \times 0.25 = \frac{0.1}{0.16}$$

0.16

(3)

(Total for Question 14 is 5 marks)



15

$$a = \frac{v - u}{t}$$

$v = 9.6$ correct to 1 decimal place

$u = 3.8$ correct to 1 decimal place

$t = 1.84$ correct to 2 decimal places

Calculate the upper bound for the value of a .

Give your answer as a decimal correct to 2 decimal places.

Show your working clearly.

$$v = 9.6 \begin{array}{l} \nearrow 9.65 \\ \searrow 9.55 \end{array}$$

$$u = 3.8 \begin{array}{l} \nearrow 3.85 \\ \searrow 3.75 \end{array}$$

$$t = 1.84 \begin{array}{l} \nearrow 1.845 \\ \searrow 1.835 \end{array}$$

$$\begin{aligned} \text{Ans} &= \frac{9.65 - 3.75}{1.835} \\ &= 3.21525\dots \\ &\quad \uparrow \\ &\quad (2\text{dp}) \end{aligned}$$

3.22

(Total for Question 15 is 3 marks)



16 The diagram shows the positions of three ships, A , B and C .

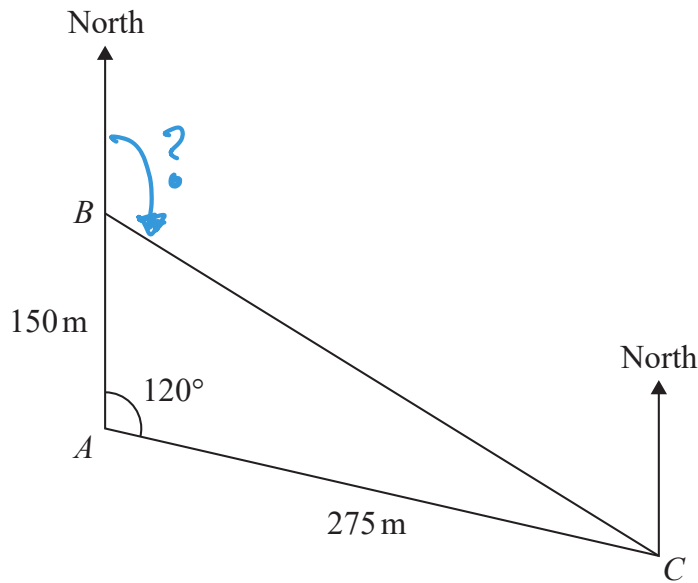


Diagram NOT accurately drawn

Ship B is due north of ship A .

The bearing of ship C from ship A is 120°

Calculate the bearing of ship C from ship B .

Give your answer correct to the nearest degree.

$$BC^2 = 150^2 + 275^2 - 2 \times 150 \times 275 \times \cos 120$$

$$= 139\,375$$

$$BC = \sqrt{139\,375} = 25\sqrt{223}$$

angle $\hat{A}BC$

$$\frac{\sin B}{275} = \frac{\sin 120}{25\sqrt{223}}$$

$$B = \sin^{-1} \left[\frac{\sin 120 \times 275}{25\sqrt{223}} \right]$$



$$= 39.637\dots$$

$$\begin{aligned}\therefore \text{bearing} &= 180 - 39.637\dots \\ &= 140.36\dots \\ &\quad \uparrow \\ &\quad \text{nearest degree}\end{aligned}$$

140 °

(Total for Question 16 is 5 marks)



17 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

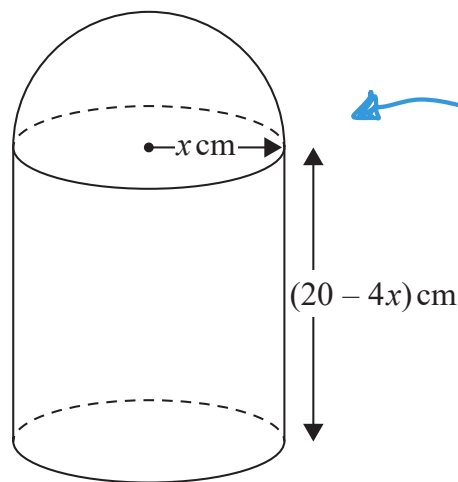


Diagram NOT accurately drawn

Volume = $\frac{1}{2} \times \frac{4}{3} \pi r^3$

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

The volume of **S** is V cm³ where $V = \frac{1}{3} \pi y$

Find the maximum value of y .
Show clear algebraic working.

$$V = \frac{1}{2} \times \frac{4}{3} \pi x^3 + \pi x^2 \times (20 - 4x)$$

$$= \frac{4}{6} \pi x^3 + 20\pi x^2 - 4\pi x^3$$

$$= \left(\frac{2}{3}\pi - 4\pi\right) x^3 + 20\pi x^2 \Rightarrow \frac{1}{3}\pi y$$

$$\times 3 = (2 - 12)x^3 + 60x^2 = y$$

$$\therefore y = 60x^2 - 10x^3$$



$$\frac{dy}{dx} = 120x - 30x^2$$

$$120x - 30x^2 = 0$$

$$30x^2 = 120x$$

$$x = \frac{120}{30} = 4$$

$$\begin{aligned}\therefore y &= 60 \times 4^2 - 10 \times 4^3 \\ &= 320\end{aligned}$$

320

(Total for Question 17 is 5 marks)

- 18 Given that $(8 - \sqrt{x})(5 + \sqrt{x}) = y\sqrt{x} + 21$ where x is a prime number and y is an integer, find the value of x and the value of y . Show each stage of your working clearly.

$$= 40 + 8\sqrt{x} - 5\sqrt{x} - x$$

$$= 40 - x + 3\sqrt{x} \quad \text{c.f.} \quad 21 + y\sqrt{x}$$

$$40 - x = 21$$

$$y = 3$$

$$x = 40 - 21$$

$$= 19$$

$$x = 19$$

$$y = 3$$

(Total for Question 18 is 3 marks)



19 Solve the simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$

$$x + 2y - 1 = 0$$

Show clear algebraic working.

$$x = 1 - 2y$$

$$\begin{aligned}x^2 &= (1 - 2y)(1 - 2y) \\ &= 1 - 4y + 4y^2\end{aligned}$$

$$1 - 4y + 4y^2 - 9y - (1 - 2y) = 2y^2 - 12$$

$$1 - 4y + 4y^2 - 9y - 1 + 2y - 2y^2 + 12 = 0$$

$$2y^2 - 11y + 12 = 0$$

$$(2y - 3)(y - 4) = 0$$

↓

$$y = \frac{3}{2}$$

↓

$$y = 4$$

$$x = 1 - 2\left(\frac{3}{2}\right)$$

$$= -2$$

$$\begin{aligned}y &= 1 - 2 \times 4 \\ &= -7\end{aligned}$$

$$x = -2$$

$$y = \frac{3}{2}$$

$$x = -7$$

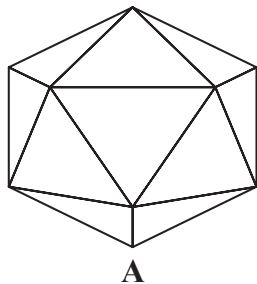
$$y = 4$$

(Total for Question 19 is 5 marks)

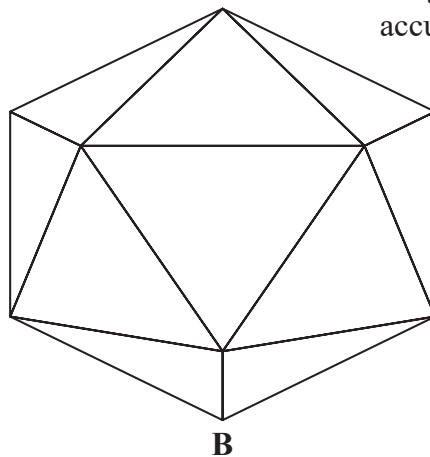


20 A and B are two similar solids.

Diagram NOT
accurately drawn



A



B

A has a volume of 1836 cm^3

B has a volume of 4352 cm^3

B has a total surface area of 1120 cm^2

Work out the total surface area of A.

$$\text{Volume scale factor} = \frac{4352}{1836}$$

$$\text{length SF} = \sqrt[3]{\frac{63}{27}} = \frac{4}{3}$$

$$\text{area SF} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$\begin{aligned} \text{S. Area of A} &= 1120 \div \frac{16}{9} \\ &= 630 \end{aligned}$$

..... 630 cm^2

(Total for Question 20 is 3 marks)



P 6 6 2 9 7 A 0 2 3 2 8

21 A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

$$\begin{array}{cc} x & y \\ -9-3 = -12 & 15 \end{array}$$

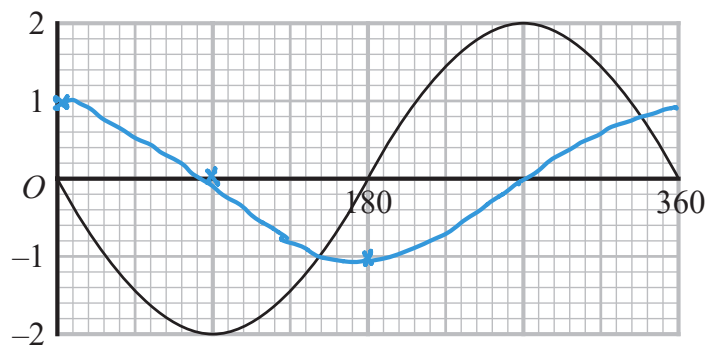
(.....-12....., 15.....)

(ii) $y = \frac{1}{3}f(x)$

$$\begin{array}{cc} x & y \\ -9 & 15 \div 3 \\ & 5 \end{array}$$

(.....-9....., 5.....)
(2)

The graph of $y = a \cos(x + b)^\circ$ for $0 \leq x \leq 360$ is drawn on the grid below.



Given that $a > 0$ and that $0 < b < 360$

(b) find the value of a and the value of b .

$$\begin{array}{l} a = 2 \\ b = 90 \end{array}$$

$a =$ 2

$b =$ 90

(2)

(Total for Question 21 is 4 marks)



22 The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leq 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

$$y = (x - 4)^2 - 16 + 5$$
$$= (x - 4)^2 - 11$$

$$y + 11 = (x - 4)^2$$

$$\sqrt{y + 11} = x - 4$$

$$x = 4 + \sqrt{y + 11}$$

given $x \leq 4$

$$x = 4 - \sqrt{y + 11}$$

$$\therefore f^{-1}(x) = 4 - \sqrt{x + 11}$$

$$f^{-1}(x) = 4 - \sqrt{x + 11}$$

(Total for Question 22 is 3 marks)



23 OAB is a triangle.

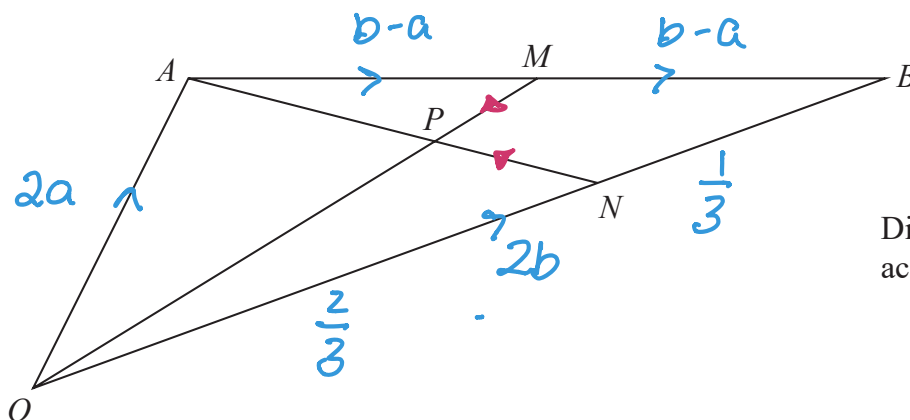


Diagram NOT accurately drawn

$$\vec{OA} = 2\mathbf{a} \text{ and } \vec{OB} = 2\mathbf{b}$$

M is the midpoint of AB .

N is the point on OB such that $ON:NB = 2:1$

P is the point on AN such that OPM is a straight line.

Use a vector method to find $OP:PM$

Show your working clearly.

$$\begin{aligned} \vec{OP} &= \vec{OA} + \lambda \vec{AN} \\ &= 2\mathbf{a} + \lambda \left(\frac{4}{3}\mathbf{b} - 2\mathbf{a} \right) \end{aligned}$$

$$\vec{AB} = 2\mathbf{b} - 2\mathbf{a}$$

$$\vec{AM} = \vec{MB} = \mathbf{b} - \mathbf{a}$$

$$\begin{aligned} \vec{OM} &= 2\mathbf{a} + \mathbf{b} - \mathbf{a} \\ &= \mathbf{a} + \mathbf{b} \end{aligned}$$

$$\vec{NA} = -\frac{2}{3} \times 2\mathbf{b} + 2\mathbf{a} = -\frac{4}{3}\mathbf{b} + 2\mathbf{a}$$

$$\vec{OP} = \mu(\mathbf{a} + \mathbf{b})$$

$$\vec{OP} = \frac{4}{3}\mathbf{b} + \lambda \left(-\frac{4}{3}\mathbf{b} + 2\mathbf{a} \right)$$

$$\mu\mathbf{a} + \mu\mathbf{b} = \frac{4}{3}\mathbf{b} - \frac{4\lambda}{3}\mathbf{b} + 2\lambda\mathbf{a}$$

$$= \left(\frac{4}{3} - \frac{4\lambda}{3} \right) \mathbf{b} + 2\lambda\mathbf{a}$$

$\times 3$

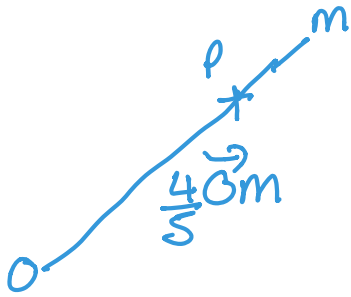
$$3\mu\mathbf{a} + 3\mu\mathbf{b} = (4 - 4\lambda)\mathbf{b} + 6\lambda\mathbf{a}$$

$$3\mu = 6\lambda$$

$$3\mu = 4 - 4\lambda$$

$$6\lambda = 4 - 4\lambda$$





$$10r = 4$$

$$r = \frac{4}{10} = \frac{2}{5}$$

$$\therefore OP : PM \\ 4 : 1$$

4 : 1

(Total for Question 23 is 6 marks)

Turn over for Question 24



24 An arithmetic series has first term a and common difference d .

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d .
Show your working clearly.

$$S_{2n} = \frac{2n(2a + (2n-1)d)}{2}$$

$$= n(2a + 2nd - d)$$

$$= 2na + 2n^2d - nd$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(2a + nd - d)$$

$$= na + \frac{n^2d}{2} - \frac{nd}{2}$$

$$2na + 2n^2d - nd = 4\left(na + \frac{n^2d}{2} - \frac{nd}{2}\right)$$

$$2na + \cancel{2n^2d} - nd = 4na + \cancel{2n^2d} - 2nd$$

$$0 = 4na - 2na - 2nd + nd$$

$$= 2na - nd$$

$$d = 2a$$

$$a = \frac{d}{2}$$

$$a = \frac{d}{2}$$

(Total for Question 24 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

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